"COMPUTER-BASED SIMULATION OF AUCTIONS OF OPTION CONTRACTS AND OF FUTURES CONTRACTS IN THE COLOMBIAN WHOLESALE ELECTRICITY MARKET"

Final Report – Chapter 2

Prepared for:

Comisión de Regulación de Energía y Gas (CREG)

The World Bank

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PART II: MODELING OF THE COLOMBIAN ELECTRICITY MARKET

1. INTRODUCTION

1.1. Assumptions

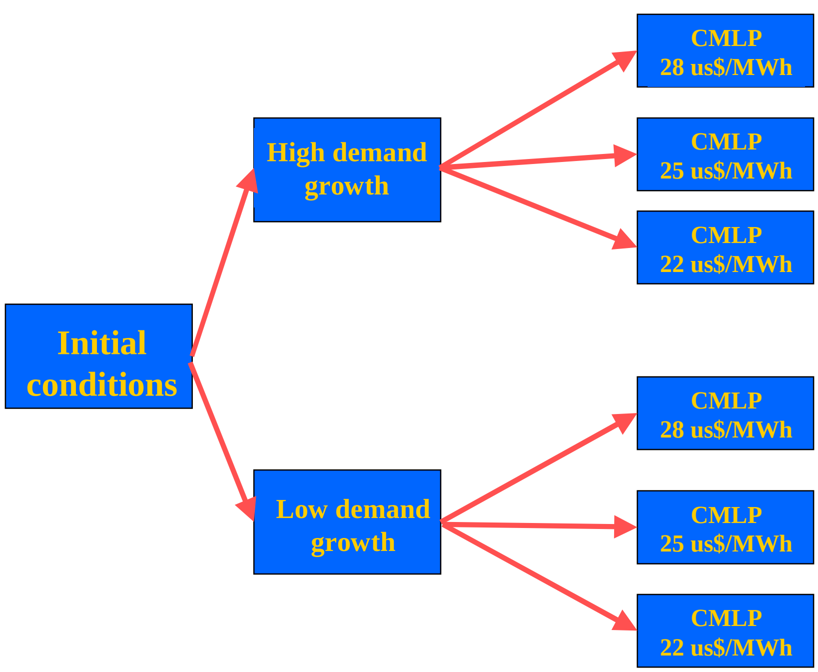
The modeling of the market was carried out using the current commercial and operational rules of the wholesale market.

The information for carrying out the market simulations was supplied by the CREG and matches with the data base used for developing market forecasts by the "Centro Nacional de Despacho" CND-ISA (market operator).

As will be shown ahead, the simulations run with the SDDP with the variables selected from the data base provided by the CND-ISA, produce results that can be adapted to evaluate the price of the options.

Given that uncertainty increases along the simulation period (2003-2010), we simulated market conditions under six different scenarios. Though these are explained in detail in section 3 of this chapter of the Report, Figure 1 provides a brief description of the six scenarios considered.

Figure 1



The current market conditions were used as the initial conditions for the simulations, and we assumed that in the short term the main source of volatility is the demand growth. Two rates of demand growth were used and an even probability of 50% of occurrence was given to each one. Thus, a "high demand growth" and a "low demand growth" setting were considered.

In the long term, uncertainty with the Long Term Marginal Cost (LTMC) increases volatility. The LTMC includes many sources of uncertainty such as fuel prices, capital cost of new plants and the rate of return expected by investors. Hence, for each of the settings three (3) LTMC were used. We assumed that each of these LTMC are equally probable.

Therefore six market simulations were performed. In each market simulation 22 hydrological records were used. It means that for each month, 132 energy prices were obtained (6 scenarios x 22 hydrological records).

The estimations of the value of options for different exercise prices, average energy prices and volatility levels were computed using the results of these simulations.

Assuming different probabilities at each scenario it is possible to obtain alternative estimations of options values, volatility and average energy prices.

In the next sections we describe the main data and the simulation process.

2. MARKET DATA

2.1. DEMAND

2.1.1. EXPECTED ENERGY DEMAND GROWTH

The following table presents the demand growth that we assumed for both settings: the "high demand growth" setting and the "low demand growth" setting. The columns named as Base Case represent the "low demand growth" setting and the figures correspond to the expected demand growth according to the ""Long Term Energy Plan 2002 - 2007"" made by ISA.

On the other hand, the columns named as High Case represent the "high demand growth" setting and the figures were estimated assuming a rate of growth of prospective demand of 5.5%.

COLOMBIAN
Forecast of the Internal Electric Demand

Year	Base Case		High Case	
	Gross Demand [GWh/yr]	Growth rate [%]	Gross Demand [GWh/yr]	Growth rate [%]
2001	43379		43379	
2002	44761	3.19%	44761	3.19%
2003	46534	3.96%	47223	5.50%
2004	48232	3.65%	49820	5.50%
2005	49864	3.38%	52560	5.50%
2006	51523	3.33%	55451	5.50%
2007	53303	3.45%	58501	5.50%
2008	55161	3.49%	61719	5.50%
2009	57161	3.63%	65113	5.50%
2010	59213	3.59%	68694	5.50%
2011	61349	3.61%	72472	5.50%
2012	63562	3.61%	76458	5.50%

Note: Year 2001 : Real demand.

The following Table presents the expected monthly energy distribution of 2003:

Expected montly energy distribution [%]

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dic
2003	8.23%	7.70%	8.45%	8.10%	8.51%	8.02%	8.50%	8.52%	8.36%	8.58%	8.37%	8.67%

2.1.2. DEMAND REPRESENTATION

To simulate the economic dispatch of generation, the demand was represented on a monthly basis in different blocks and according to the characteristic load shape of Colombia.

- Block 1: **3.36 %** (Avg. 25 hrs/month)
- Block 2: **24.16 %** (Avg. 176 hrs/month)
- Block 3: **24.16 %** (Avg. 176 hrs/month)
- Block 4: **24.16 %** (Avg. 176 hrs/month)
- Block 5: **24.16 %** (Avg. 176 hrs/month)

The Table below shows the different factor applied to each hour block.

Factor of distribution of the energy in each block [%]

Block	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dic
Block 1	5.04%	4.80%	4.98%	5.02%	4.94%	4.82%	4.98%	4.88%	4.96%	5.04%	5.00%	5.16%
Block 2	27.7%	27.8%	27.6%	27.7%	27.6%	27.6%	27.6%	27.6%	27.7%	27.7%	27.7%	27.7%
Block 3	25.0%	25.1%	25.0%	25.0%	25.0%	25.0%	25.0%	25.0%	25.0%	25.0%	25.0%	25.0%
Block 4	23.3%	23.3%	23.3%	23.3%	23.3%	23.3%	23.3%	23.3%	23.3%	23.3%	23.3%	23.2%
Block 5	19.0%	19.0%	19.1%	19.0%	19.1%	19.2%	19.1%	19.2%	19.1%	19.0%	19.0%	18.9%
Total	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*

2.2. *GENERATION: EXISTENT POWER-PLANTS IN COLOMBIA*

The total installed generation capacity is about 13.000 MW (including small hydro plants and cogeneration). The following tables identify the existing thermal and hydro power plants.

EXISTING PLANTS			
Type	Install Capacity	Effective Capacity	
	[MW]	[MW]	[%]
Hydro Plants	8730	8618	70.8
Thermal Plants	4607	3560	29.2
Total	**13337**	**12177**	100.0

EXISTING HYDRO PLANTS				
Name	Type	Install Capacity	Effective Capacity	
		[MW]	[MW]	[%]
URRA	Hydro	340	329	3.8
GUATAPE	Hydro	560	560	6.5
JAGUAS	Hydro	170	170	2.0
PLAYAS	Hydro	201	201	2.3
SAN CARLOS	Hydro	1240	1240	14.4
CALDERAS	Hydro	24	24	0.3
TRONERAS	Hydro	40	40	0.5
GUADALUPE 3	Hydro	270	270	3.1
GUADALUPE 4	Hydro	225	225	2.6
LA TASAJERA	Hydro	306	306	3.6
RIOGRANDE 1	Hydro	75	75	0.9
INSULA	Hydro	18	18	0.2
ESMERALDA	Hydro	30	30	0.3
SAN FRANCISC	Hydro	135	135	1.6
GUAVIO	Hydro	1150	1150	13.3
PARAISO	Hydro	270	270	3.1
LA GUACA	Hydro	310	310	3.6
SALTO	Hydro	64	63	0.7
LAGUNETA	Hydro	36	36	0.4
COLEGIO	Hydro	150	150	1.7
CHIVOR	Hydro	1078	1000	11.6
PRADO	Hydro	44	44	0.5
PRADO4	Hydro	5	5	0.1
FLORIDA	Hydro	26	26	0.3
RIOMAYO	Hydro	21	21	0.2
BETANIA	Hydro	540	540	6.3
ALTOANCHICAY	Hydro	365	365	4.2
BAJOANCHICAY	Hydro	74	74	0.9
CALIMA	Hydro	120	120	1.4
SALVAJINA	Hydro	285	285	3.3
PORCE II	Hydro	405	405	4.7
Subtotal Hydro		**8577**	**8487**	98.5
Small Hydro	Hydro	153	130	1.5
Total Hydro		**8730**	**8618**	100.0

EXISTING THERMAL PLANTS				
Name	**Type**	**Install Capacity**	**Effective Capacity**	
		[MW]	**[MW]**	**[%]**
BARRANQUILL3	Thermal	63	46	1.3
BARRANQUILL4	Thermal	63	32	0.9
CARTAGENA 1	Thermal	65	52	1.5
CARTAGENA 2	Thermal	44	28	0.8
CARTAGENA 3	Thermal	70	53	1.5
GUAJIRA 1	Thermal	151	105	2.9
GUAJIRA 2	Thermal	151	123	3.4
FLORES 1	Thermal	150	133	3.7
PALENQUE 3	Thermal	14	12	0.3
BARRANCA 1	Thermal	11	7	0.2
BARRANCA 2	Thermal	12	11	0.3
BARRANCA 3	Thermal	54	4	0.1
BARRANCA 4	Thermal	30	6	0.2
BARRANCA 5	Thermal	20	3	0.1
PAIPA 1	Thermal	28	25	0.7
PAIPA 2	Thermal	68	56	1.6
PAIPA 3	Thermal	68	60	1.7
TASAJERO 1	Thermal	155	145	4.1
ZIPAEMG2	Thermal	34	8	0.2
ZIPAEMG3	Thermal	62	0	0.0
ZIPAEMG4	Thermal	62	60	1.7
ZIPAEMG5	Thermal	63	21	0.6
YUMBO3	Thermal	29	0	0.0
FLORES 2	Thermal	99	84	2.4
TEBSAB	Thermal	750	695	19.5
TERMODORADA1	Thermal	51	45	1.3
TERMOSIERRA	Thermal	448	358	10.1
MERILECTRICA	Thermal	154	131	3.7
TPIEDRAS	Thermal	3	3	0.1
TERMOVALLE 2	Thermal	201	152	4.3
PAIPA 4	Thermal	150	138	3.9
FLORES 3	Thermal	149	135	3.8
COROZO 1	Thermal	54	0	0.0
CADAFE 1	Thermal	36	0	0.0
PROELECTRIC1	Thermal	42	30	0.9
PROELECTRIC2	Thermal	42	30	0.9
EMCALI	Thermal	231	155	4.4
TERMOCENTRO	Thermal	279	237	6.7
CANDELARIA1	Thermal	150	128	3.6
CANDELARIA2	Thermal	250	213	6.0
COG_PROENCA	Thermal	4	2	0.0
AUTOPROD_OXI	Thermal	39	26	0.7
COG_INCAUCA	Thermal	9	5	0.1
Total Thermal		**4607**	**3560**	100.0

2.3. GENERATION EXPANSION PLAN

As the demand grows, it will be necessary to start-up new plants to keep the supply/demand balanced within an economic rationality.

2.3.1. SHORT AND MEDIUM TERM GENERATION PROJECTS

Future generation projects considered for short and medium term are described in this section.

GENERATION EXPANSION PLAN

Commitment Date	Plant	Type	Nr of Units	Install Capacity	Total Install Capacity	Observations
September-02	Miel II	Hydro	1	125	125	
October-02	Miel II	Hydro	2	125	250	
December-02	Miel II	Hydro	3	125	375	

2.3.1. LONG TERM MARGINAL COST

On account of the expected electricity demand growth in any competitive electricity market, the incorporation of new generation equipment to meet the demand under certain quality conditions should be considered in the long run. Current installed capacity would be insufficient for such sustained growth.

Consequently, certain criteria should be established with the purpose of determining the main characteristics of such future plants (type, capacity, etc.) as well as the expected commissioning date.

Thus, in order to forecast long term prices of energy, it is necessary to underline the assumptions about how decisions on investments in generation will be made. Some assumptions were posed about how private investors are really encouraged to develop new power plants to supply the demand growth and maintain the system reliability within limits accepted by consumers and the government.

- The most common approach is to assume that the new investments will be encouraged by market prices. When prices are high enough to produce an attractive rate of return, new plants will be installed. When prices are below those necessary to produce the expected rate of return, no new plants are installed. The price that sets the limit between attractive and non attractive rates of return is the long term marginal cost (LTMC).

  For simulation purposes we assumed that new plants are installed whenever the average short term marginal cost raises above the LTMC. The basic hypothesis underlying the LTMC concept is that investments in new generation projects in the long run would be made exclusively under market risk conditions. This means that such projects should be financed yielding a reasonable rate of return based on revenues obtained from the sale of its production at the wholesale market.

  No matter which generation installed capacity expansion alternative is finally adopted in Colombia for the long run, the necessary condition to be met by all future projects, from the investor's standpoint, is the economic rationality that allows for a definition of the opportunity when the new power-plants will be incorporated and the short-term energy price (STMC) will be within the necessary range for investment to be feasible from the financial standpoint.

To simplify the analysis we assumed that power plants will sell all the energy that they produce to the spot market. Thus, revenues can be estimated from the market simulations. Although it is common to assume that in competitive markets the price of forward contracts tend to average the spot price, recent papers show, following experience and theory from liquid commodity markets, that prices of forwards should be lower than the average spot price.

The LTMC was estimated based on:

- Expected rate of return for new investments. It may be computed using the WACC approach. The WACC approach considers properly the risks related to the country, regulatory and the systematic risk related to generation activity, as well as the financing of new plants.

- The capital cost of the most convenient type(s) of plant(s) in Colombia for expanding generation. Large scale combined cycles burning gas was adopted as the typical expansion plant.

- The costs of gas used by the expansion plants.

- The expected stream of revenues of new power plants that operate under the spot market rules.

- The "expansion plants" must be profitable for an internal rate of return (IRR) equal to the WACC with a reasonable equity/debt capital structure.

  As mentioned at the beginning of this chapter, to take into consideration uncertainty a basic LTMC was computed using the best estimation of the more relevant parameters. Two sensitivities were computed, assuming deviations in the main parameters. Finally we considered three different scenarios for the long term marginal cost, those values are **22**, **25** y **28 $/MWh**.

2.3.2. LONG TERM GENERATION EXPANSION :

For simulating the long term generation expansion, we assumed that the expansion will be done on the basis of combined cycle burning Natural Gas plants of gross capacity equal to **300 MW**. In the table below the expansion scheduled for each scenario is shown.

Date Incomes	Gross Capacity [MW]					
	Demand 3%			Demand 5%		
	LTMC = 22 Case # 1	LTMC = 25 Case # 2	LTMC = 28 Case # 3	LTMC = 22 Case # 4	LTMC = 25 Case # 5	LTMC = 28 Case # 6
2005	0	0	0	300	300	0
2006	0	0	0	300	0	300
2007	0	0	0	300	300	0
2008	300	300	0	600	600	600
2009	300	0	300	300	300	300
2010	300	300	300	600	600	600
2011	300	600	300	600	600	600
2012	0	0	0	0	300	300
Total	**1200**	**1200**	**900**	**3000**	**3000**	**2700**

Note : Future projects **300 MW** gross capacity, Combined Cycle.

2.4. COST OF NON SUPPLIED ENERGY

For the economic dispatch simulation, the "rationing function" was modeled in two steps as it is shown in the table below:

Rationing Level (% of Total Demand)	**Cost of NSE**
1.5 %	**176** $US/MWh
98.5 %	**320** $US/MWh

3. SCENARIOS

Six different scenarios of simulation were defined. Such scenarios were configured as the combination of two demand scenarios (high demand growth setting and low demand growth setting) and three Long Term Marginal Cost. The following table relates each scenario we used, with the demand setting considered and the LTMC:

Scenario	**Case # 1**	**Case # 2**	**Case # 3**	**Case # 4**	**Case # 5**	**Case # 6**
Demand	Base Case	Base Case	Base Case	High Case	High Case	High Case
LTMC [US$/MWh]	22.00	25.00	28.00	22.00	25.00	28.00

4. SIMULATION MODEL

4.1. THE SDDP MODEL

The SDDP (Stochastic Dual Dynamic Programming) hydrothermal dispatch model was used to simulate the evolution of the Wholesale Electricity Market of Colombia.

The SDDP model allows for the representation of the joint dispatch of the hydrothermal installed capacity, taking into account variable fuel costs as well as operation and maintenance cost of each plant. For that purpose, the model estimates the **"value of water"** contained in dams based on a stochastic dual dynamic programming algorithm, that optimizes the water resources use, minimizing the fuel supply cost in both short and long.

The electricity system simulation is carried out for a pre-defined set of hydrological records that are obtained on the basis of historical inflow data. The main results obtained (system marginal costs or wholesale prices, generator revenues, etc.) are an average of the values obtained from the respective variables in each simulated hydrological record.

Detail information about the SDDP model is included in Appendix III at the end of this chapter.

4.2. ENERGY PRICE IN THE SPOT MARKET.

In the Colombian Wholesale Electric Market the restrictions of transport (congestion and losses) are not depicted for the determination of the Spot prices.

The energy price in the spot market was calculated for every month and demand bloc until the year 2012.

4.3. SIMULATION PROCEDURE

As mentioned previously, six possible future scenarios were proposed. The following steps were followed for each of such scenarios in order to obtain the necessary information.

- ✓ A simulation was performed (for each scenario) with the purpose of determining the prospective evolution of the Colombian Wholesale Electricity Market (WEM). We assumed that long term generation expansion will be done on the basis of Combined Cycles using Natural Gas, given that with this technology, the system reaches the minimum LTMC.
- ✓ Once the expansion plan for a certain scenario was defined the following information was obtained from the SDDP model:
    - ➢ each power-plant dispatch,
    - ➢ the system prices for each hydrological series, each month of the period under analysis and each of the 4 hourly blocks under consideration.
- ✓ Based on the obtained data, the power plant's revenues per year and per hydrological series could be determined and, consequently, the annual mean value expected from such revenues.
- ✓ We assumed that the characteristics of the hydraulic power stations and of the thermal power stations are the same ones than those considered by ISA in the "Long Term Energy Plan 2002 - 2007." The characteristics of inflow water of each dam were simulated according the historical production (1998-2001).

- ✓ No electricity exchanges were considered with neighbor countries.

4.4. EXERCISE PRICES

The prices of options were estimated for each exercise price as the mathematical expectation of the price, whenever the price is above of every exercise price.

This means that:

$$OPpe\text{-}t = \sum_{i=1,5} \sum_{j=1,n} PStijpe$$

Where:

OPpe-t: price of an option for exercise price pe, period t (months since Jan 2003 to Dec 2010)

.i: index for demand blocks

.j: index for price realizations (one for every scenario and hydrological record)

PMtij: energy prices obtained from market simulation, for period t, price realization j and block i

PStijpe: defined as

if PMtij > pe PStijpe = PMtij – pe

if PMtij <= pe PStijpe = 0

The following steps were carried out for each month of the simulation horizon:

- ✓ For every month, the weighted average price was obtained after combining the prices obtained in each of the six scenarios with the weighted probability assigned to each scenario. As we was mentioned previously, scenarios equally probable were assumed.
- ✓ For a given exercise price, the option price was determined.
- ✓ The same procedure was carried out for every exercise price.

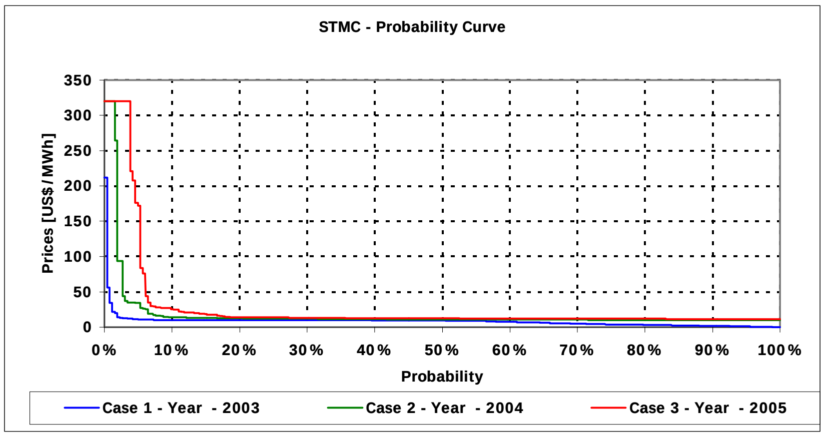
5. SIMULATION RESULTS

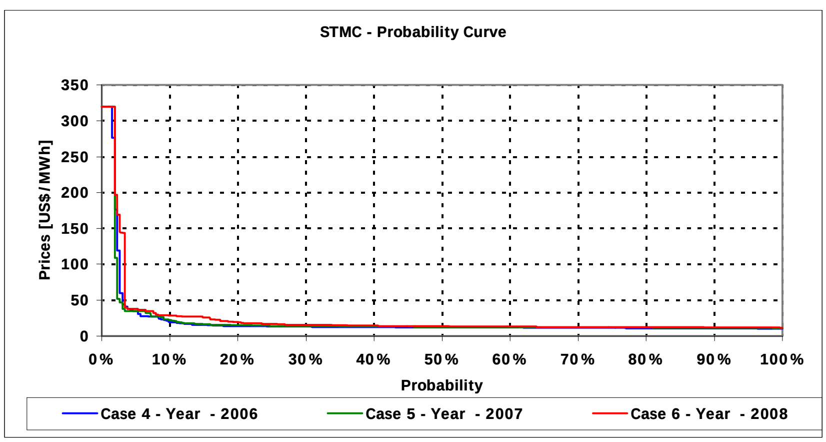
5.1. ENERGY PRICES

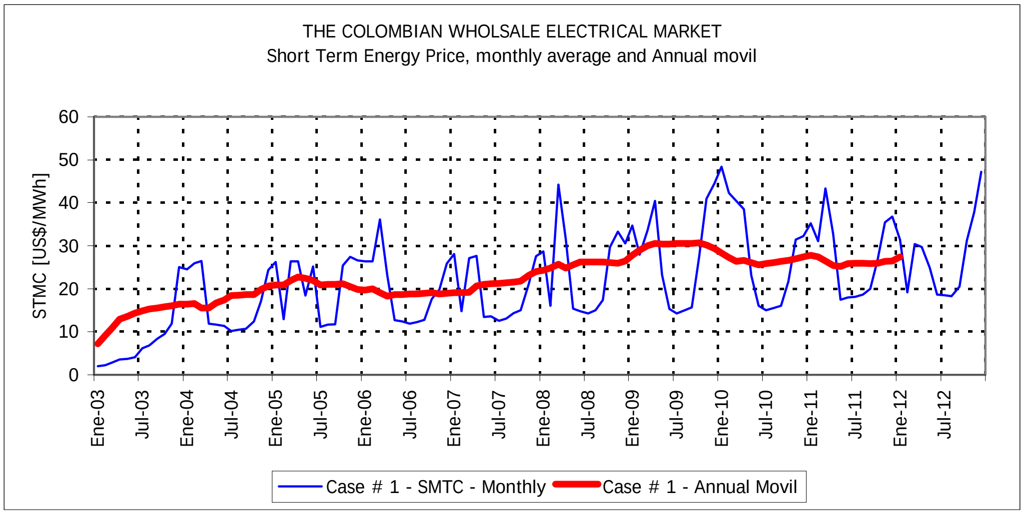
The following table and figures show the average energy prices obtained in each scenario. Averages are computed on hydrology. Tables and figures show the energy prices as the average value for each month of the simulated period, the annual moving average value and the probability price curve.

The detailed information about energy prices can be found in Appendix IV at the end of this chapter.

Average STMC in the Colombian WEM [US$/MWh]						
Year	Case # 1	Case # 2	Case # 3	Case # 4	Case # 5	Case # 6
2003	7.21	6.85	7.06	8.46	8.78	8.55
2004	16.38	18.45	17.91	19.27	18.41	19.23
2005	20.82	21.86	21.86	28.69	24.60	22.13
2006	19.75	19.15	20.35	28.02	25.28	21.36
2007	19.02	20.71	20.40	27.00	25.38	21.72
2008	24.29	24.03	21.76	26.96	24.83	23.10
2009	27.72	25.08	21.51	26.86	25.28	23.06
2010	28.32	25.26	22.88	26.22	24.13	21.17
2011	27.77	24.84	23.05	26.55	24.20	22.23
2012	27.40	24.25	22.54	28.83	27.16	21.33



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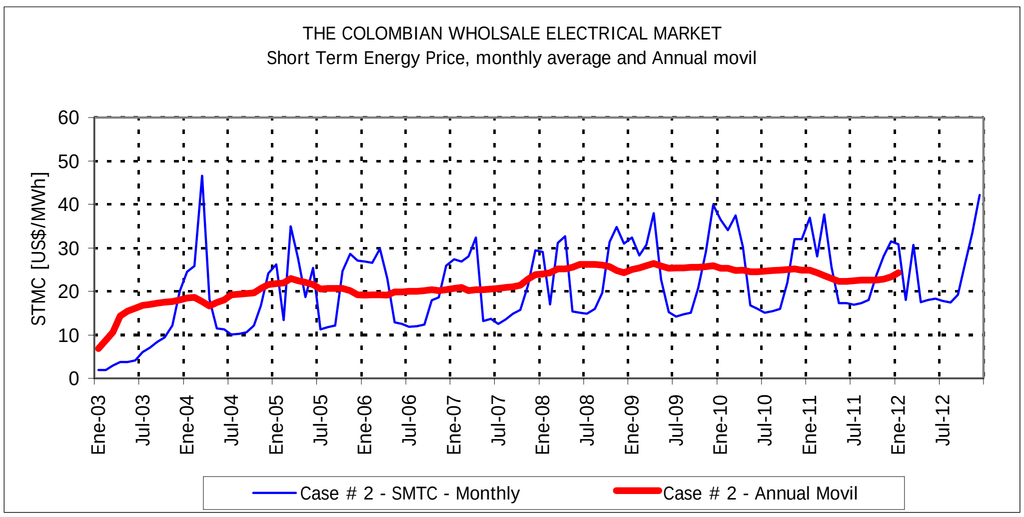
Case # 1 - SMTC - Monthly
Case # 1 - Annual Movil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Case # 2 - SMTC - Monthly
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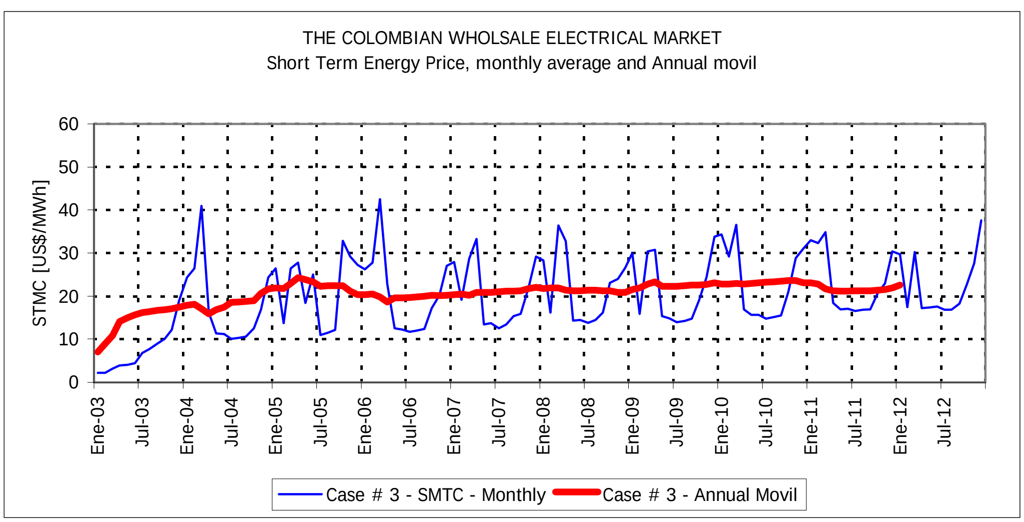
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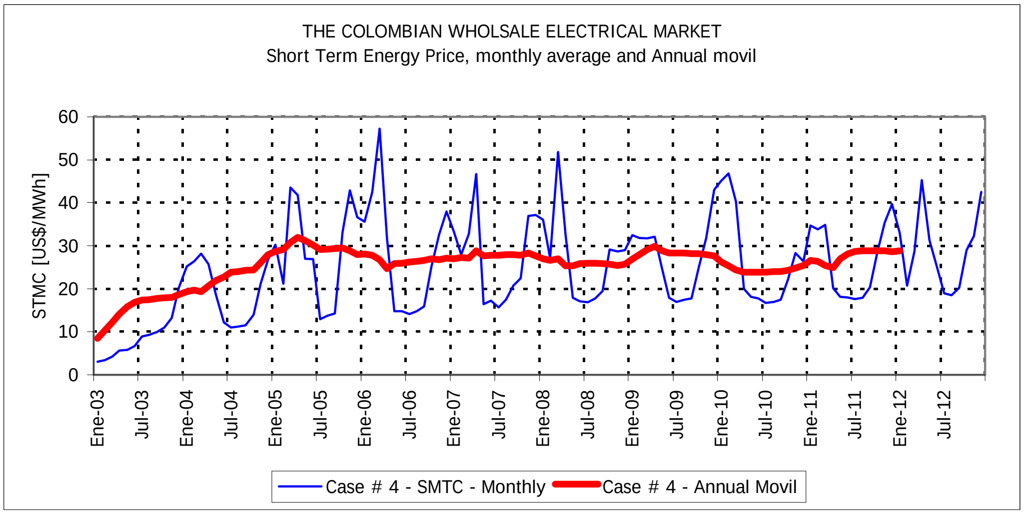
Case # 3 - SMTC - Monthly
Case # 3 - Annual Movil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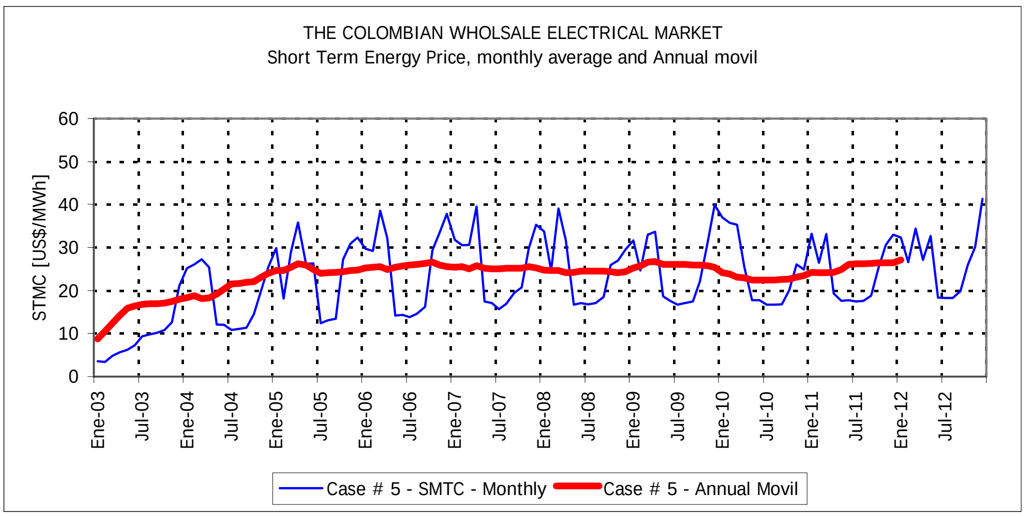
Case # 4 - SMTC - Monthly
Case # 4 - Annual Movil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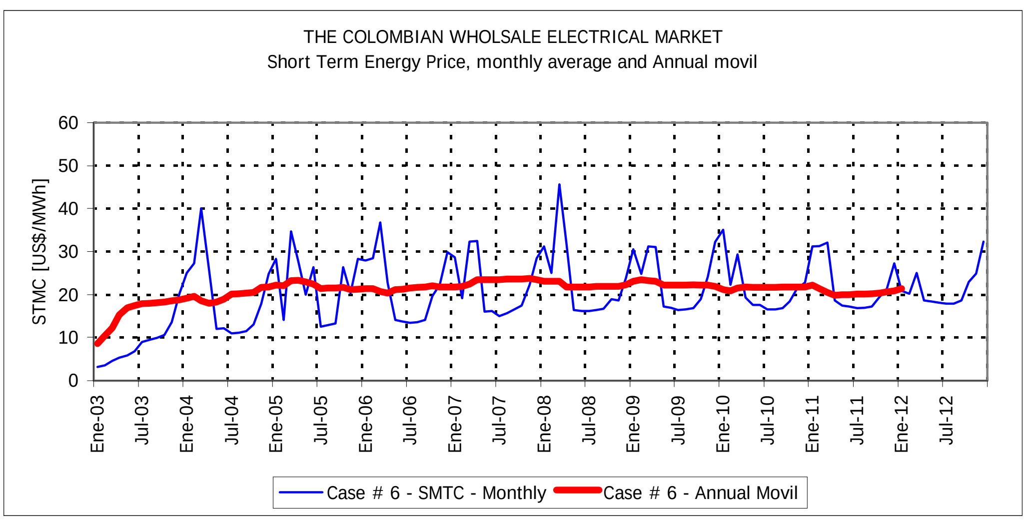
Case # 5 - SMTC - Monthly
Case # 5 - Annual Movil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Jul-07
Ene-08
Jul-08
Ene-09
Jul-09
Ene-10
Jul-10
Ene-11
Jul-11
Ene-12
Jul-12
Case # 6 - SMTC - Monthly
Case # 6 - Annual Movil

5.2. *OPTIONS VALUE ESTIMATIONS*

The following tables show the result for each semester of the period under study. An Excel attached file includes the monthly result and more options value for different exercise prices.

The following table shows when the average value of the prices is higher to each exercise price, for each semester.

Means considering the six scenarios

Average Energy Prices for each cut price [US$/MWh]

			Cut Price [US$/MWh]									
Months of the period		6	Cut#01	Cut#02	Cut#03	Cut#04	Cut#05	Cut#06	Cut#07	Cut#08	Cut#09	Cut#10
Period	Initial	Final	0.0	10.0	20.0	25.0	30.0	50.0	100.0	150.0	200.0	300.0
1	Ene-03	Jun-03	4.05	11.60								
2	Jul-03	Dic-03	11.53	16.95	72.11	85.52	106.32	183.57	224.70	224.70	239.66	319.86
3	Ene-04	Jun-04	21.84	25.11	154.12	162.75	181.10	287.64	287.64	293.75	319.86	319.86
4	Jul-04	Dic-04	14.77	15.30	78.15	85.52	104.50	137.85	267.23	267.23	267.23	
5	Ene-05	Jun-05	25.87	26.61	148.35	168.20	197.10	262.66	283.78	288.02	319.86	319.86
6	Jul-05	Dic-05	20.82	20.93	133.02	164.12	190.58	215.06	240.34	256.55	298.33	319.86
7	Ene-06	Jun-06	25.29	25.37	109.01	115.15	152.32	269.25	282.83	282.83	319.86	319.86
8	Jul-06	Dic-06	19.39	19.39	61.18	79.96	100.31	145.77	199.89	253.54	282.50	319.86
9	Ene-07	Jun-07	24.70	24.70	87.35	94.83	111.41	253.15	290.08	296.55	319.86	319.86
10	Jul-07	Dic-07	20.08	20.08	56.72	72.18	83.84	141.70	224.33	261.14	305.89	319.86
11	Ene-08	Jun-08	26.37	26.37	85.34	88.91	118.90	268.95	282.23	288.80	319.86	319.86
12	Jul-08	Dic-08	21.99	21.99	54.25	74.79	109.03	156.03	180.03	228.27	289.46	312.32
13	Ene-09	Jun-09	26.74	26.74	87.54	101.51	128.18	247.19	272.80	272.80	315.58	319.86
14	Jul-09	Dic-09	23.13	23.13	51.98	75.23	103.63	158.04	191.59	253.15	312.23	319.86
15	Ene-10	Jun-10	28.49	28.49	82.00	88.03	103.66	179.75	277.00	277.00	301.54	318.39
16	Jul-10	Dic-10	20.90	20.90	53.71	75.38	99.35	141.13	198.80	235.79	295.11	319.86
17	Ene-11	Jun-11	26.34	26.34	75.93	89.92	109.86	215.56	282.59	290.70	315.94	319.04
18	Jul-11	Dic-11	23.23	23.23	53.36	76.10	100.60	151.93	188.22	243.40	298.97	314.37
19	Ene-12	Jun-12	24.95	24.95	76.00	87.58	110.63	236.83	267.08	267.08	311.15	318.52
20	Jul-12	Dic-12	25.55	25.55	44.81	64.91	90.69	151.07	175.60	255.76	289.68	319.86

The following table shows the time (%) when the prices are higher to each exercise price, for each semester.

Means considering the six scenarios

Dispatch factor for each Cut Price [%]

			Cut Price [US$/MWh]									
Months of the period		6	Cut#01	Cut#02	Cut#03	Cut#04	Cut#05	Cut#06	Cut#07	Cut#08	Cut#09	Cut#10
Period	Initial	Final	0.0	10.0	20.0	25.0	30.0	50.0	100.0	150.0	200.0	300.0
1	Ene-03	Jun-03	100.00	3.43								
2	Jul-03	Dic-03	100.00	37.67	3.65	2.89	2.13	1.01	0.77	0.77	0.51	
3	Ene-04	Jun-04	100.00	79.59	7.66	7.20	6.34	3.67	3.67	3.54	2.90	2.90
4	Jul-04	Dic-04	100.00	90.30	5.30	4.67	3.52	2.38	0.77	0.77	0.77	
5	Ene-05	Jun-05	100.00	95.64	10.25	8.86	7.35	5.22	4.72	4.60	3.58	3.58
6	Jul-05	Dic-05	100.00	99.01	6.94	5.42	4.55	3.91	3.29	2.91	2.02	1.51
7	Ene-06	Jun-06	100.00	99.48	13.13	12.26	8.61	4.30	4.04	4.04	3.02	3.02
8	Jul-06	Dic-06	100.00	100.00	12.13	8.21	5.92	3.41	2.01	1.14	0.89	
9	Ene-07	Jun-07	100.00	100.00	15.10	13.53	10.87	3.78	3.16	3.04	2.54	2.54
10	Jul-07	Dic-07	100.00	100.00	13.30	9.17	7.29	3.27	1.51	1.14	0.77	0.51
11	Ene-08	Jun-08	100.00	100.00	16.82	15.92	10.71	3.80	3.57	3.44	2.70	2.70
12	Jul-08	Dic-08	100.00	100.00	16.25	9.95	5.78	3.53	2.78	1.38	0.64	
13	Ene-09	Jun-09	100.00	100.00	15.85	13.06	9.61	4.18	3.66	3.66	2.54	2.30
14	Jul-09	Dic-09	100.00	100.00	20.09	11.28	7.10	3.92	2.89	1.52	0.89	0.76
15	Ene-10	Jun-10	100.00	100.00	19.11	17.38	13.82	6.45	3.40	3.40	2.76	2.26
16	Jul-10	Dic-10	100.00	100.00	12.26	7.28	4.84	2.89	1.51	1.01	0.51	
17	Ene-11	Jun-11	100.00	100.00	16.45	13.07	9.92	4.06	2.77	2.64	2.15	2.03
18	Jul-11	Dic-11	100.00	100.00	17.45	10.11	6.73	3.78	2.52	1.39	0.77	0.64
19	Ene-12	Jun-12	100.00	100.00	14.32	11.78	8.53	3.17	2.69	2.69	1.81	1.68
20	Jul-12	Dic-12	100.00	100.00	30.19	16.11	9.60	4.56	3.41	1.28	0.89	0.64

The following table shows the times (hours) when the prices are higher to each exercise price, for each semester.

Means considering the six scenarios

Time for which the STMC is higher than the Cut Price [hrs]

			Cut Price [US$/MWh]									
Months of the period		**6**	**Cut#01**	**Cut#02**	**Cut#03**	**Cut#04**	**Cut#05**	**Cut#06**	**Cut#07**	**Cut#08**	**Cut#09**	**Cut#10**
Period	**Initial**	**Final**	**0.0**	**10.0**	**20.0**	**25.0**	**30.0**	**50.0**	**100.0**	**150.0**	**200.0**	**300.0**
1	Ene-03	Jun-03	4344	149								
2	Jul-03	Dic-03	4416	1664	161	128	94	45	34	34	23	6
3	Ene-04	Jun-04	4344	3458	333	313	275	159	159	154	126	126
4	Jul-04	Dic-04	4416	3987	234	206	156	105	34	34	34	
5	Ene-05	Jun-05	4344	4154	445	385	319	227	205	200	155	155
6	Jul-05	Dic-05	4416	4372	306	239	201	173	145	129	89	67
7	Ene-06	Jun-06	4344	4321	570	533	374	187	175	175	131	131
8	Jul-06	Dic-06	4416	4416	535	363	262	151	89	51	39	17
9	Ene-07	Jun-07	4344	4344	656	588	472	164	137	132	111	111
10	Jul-07	Dic-07	4416	4416	587	405	322	144	67	50	34	23
11	Ene-08	Jun-08	4344	4344	731	692	465	165	155	149	117	117
12	Jul-08	Dic-08	4416	4416	718	439	255	156	123	61	28	11
13	Ene-09	Jun-09	4344	4344	688	567	417	182	159	159	110	100
14	Jul-09	Dic-09	4416	4416	887	498	314	173	128	67	39	34
15	Ene-10	Jun-10	4344	4344	830	755	600	280	148	148	120	98
16	Jul-10	Dic-10	4416	4416	541	322	214	128	67	45	22	17
17	Ene-11	Jun-11	4344	4344	715	568	431	176	120	115	93	88
18	Jul-11	Dic-11	4416	4416	771	447	297	167	111	61	34	28
19	Ene-12	Jun-12	4344	4344	622	512	370	138	117	117	79	73
20	Jul-12	Dic-12	4416	4416	1333	711	424	201	150	56	39	28

The following table shows the revenues of a unit machine (1 MW) whose production cost is equal to the exercise price. This is the indifference price of a generator for selling an option at such exercise price or selling the energy in the spot market.

Means considering the six scenarios

Incomes for 1 MW [US$]

			Cut Price [US$/MWh]									
Months of the period		**6**	**Cut#01**	**Cut#02**	**Cut#03**	**Cut#04**	**Cut#05**	**Cut#06**	**Cut#07**	**Cut#08**	**Cut#09**	**Cut#10**
Period	**Initial**	**Final**	**0.0**	**10.0**	**20.0**	**25.0**	**30.0**	**50.0**	**100.0**	**150.0**	**200.0**	**300.0**
1	Ene-03	Jun-03	17593	1728								
2	Jul-03	Dic-03	50916	28205	11610	10947	9994	8261	7640	7640	5512	1919
3	Ene-04	Jun-04	94873	86830	51322	50941	49803	45735	45735	45238	40302	40302
4	Jul-04	Dic-04	65224	61001	18287	17617	16302	14474	9086	9086	9086	
5	Ene-05	Jun-05	112379	110538	66016	64757	62875	59624	58175	57604	49578	49578
6	Jul-05	Dic-05	91941	91506	40704	39225	38307	37205	34849	33095	26551	21431
7	Ene-06	Jun-06	109860	109624	62136	61375	56968	50350	49495	49495	41902	41902
8	Jul-06	Dic-06	85626	85626	32731	29025	26281	22011	17790	12931	11018	5438
9	Ene-07	Jun-07	107297	107297	57302	55760	52586	41517	39741	39145	35504	35504
10	Jul-07	Dic-07	88673	88673	33295	29233	26996	20405	15030	13057	10400	7357
11	Ene-08	Jun-08	114551	114551	62384	61526	55289	44377	43746	43031	37424	37424
12	Jul-08	Dic-08	97108	97108	38952	32833	27803	24341	22144	13924	8105	3436
13	Ene-09	Jun-09	116159	116159	60228	57556	53451	44989	43375	43375	34714	31986
14	Jul-09	Dic-09	102142	102142	46106	37465	32540	27341	24524	16961	12177	10875
15	Ene-10	Jun-10	123761	123761	68060	66463	62196	50330	40996	40996	36185	31202
16	Jul-10	Dic-10	92294	92294	29057	24272	21261	18065	13320	10611	6492	5438
17	Ene-11	Jun-11	114421	114421	54290	51075	47350	37939	33911	33431	29382	28076
18	Jul-11	Dic-11	102584	102584	41141	34017	29878	25372	20892	14847	10165	8802
19	Ene-12	Jun-12	108383	108383	47272	44841	40933	32683	31248	31248	24581	23252
20	Jul-12	Dic-12	112829	112829	59732	46151	38453	30365	26340	14323	11298	8956

5.3. COMPARISON WITH BLACK FORMULAE

The estimation of option prices estimated using the market simulation were compared with the same values obtained using the Black formulae.

Black's formulae is used to compute the value of European options and futures. As mentioned at the end of chapter 1 of this Report, the use of this formulae requires the existence of a liquid futures market. It is not the case of Colombia. Thus, in order to use the formulae, we assumed that the forecasted average price of the spot market is a suited proxy of a futures value.

Volatility of the prices were also estimated based on the results of market simulations.

Black's formulae is as follows:

$$\mathbf{C = e^{-rt}\ [\ F \cdot N(d1) - E \cdot N(d2)\ ]}$$

Where:

C: theoretical value of an European call option on futures .

.r: risk free interest rate in the short term market

.t: time to maturity of the option in fractions of year

F: current price of futures, estimated as the average price of the spot market, forecasted with the market simulation

E: exercise price of the call option

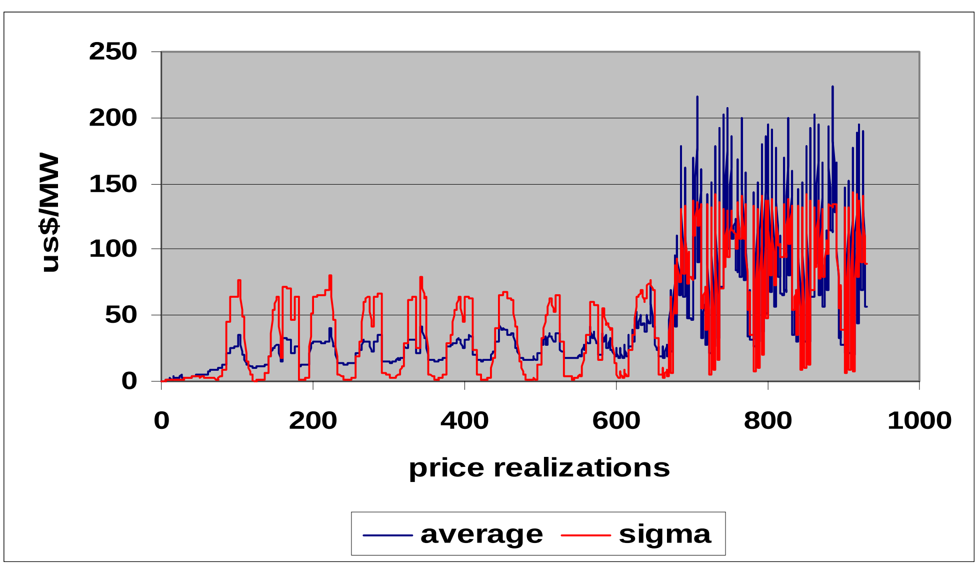
N(*): value of a normal (Gauss) distribution

σ: volatility of prices

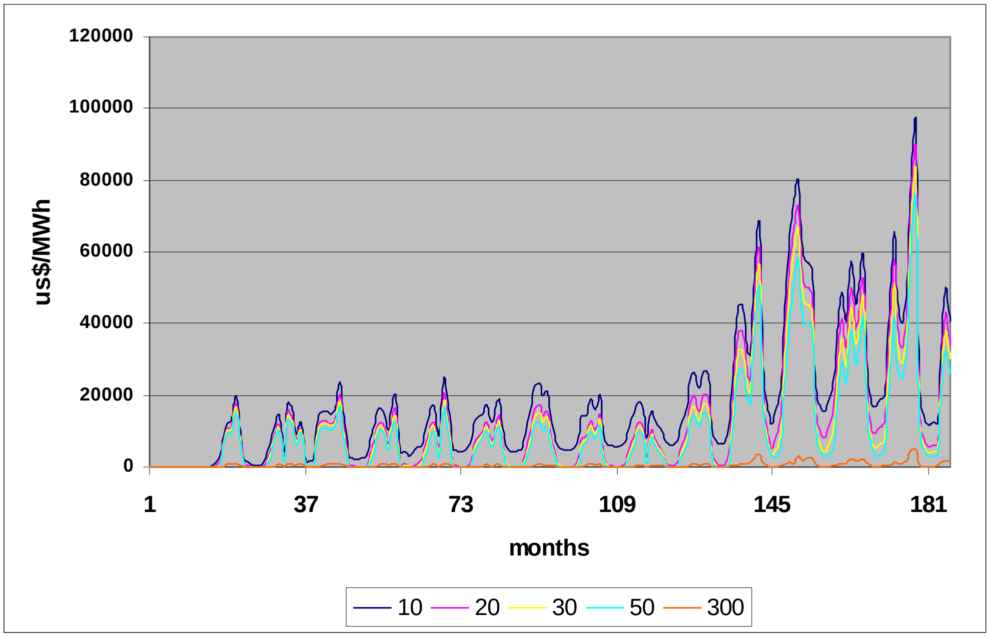
$$\mathbf{d1 = \frac{LN(F/E) + 0.5 \cdot \sigma^2 \cdot t}{\sigma \cdot \sqrt{t}}}$$

$$\mathbf{d2 = d1 - \sigma \cdot \sqrt{t}}$$

The next figure shows the forecast of average spot prices (proxy of F) and its standard deviation (proxy of σ), obtained from market simulations.



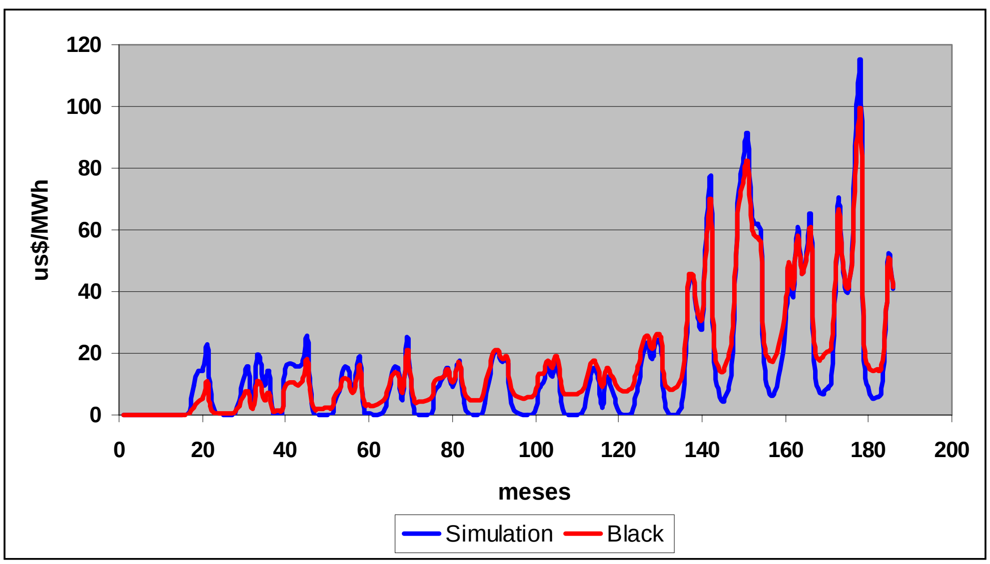
In the next figure we show the estimation of options prices for different exercise prices.



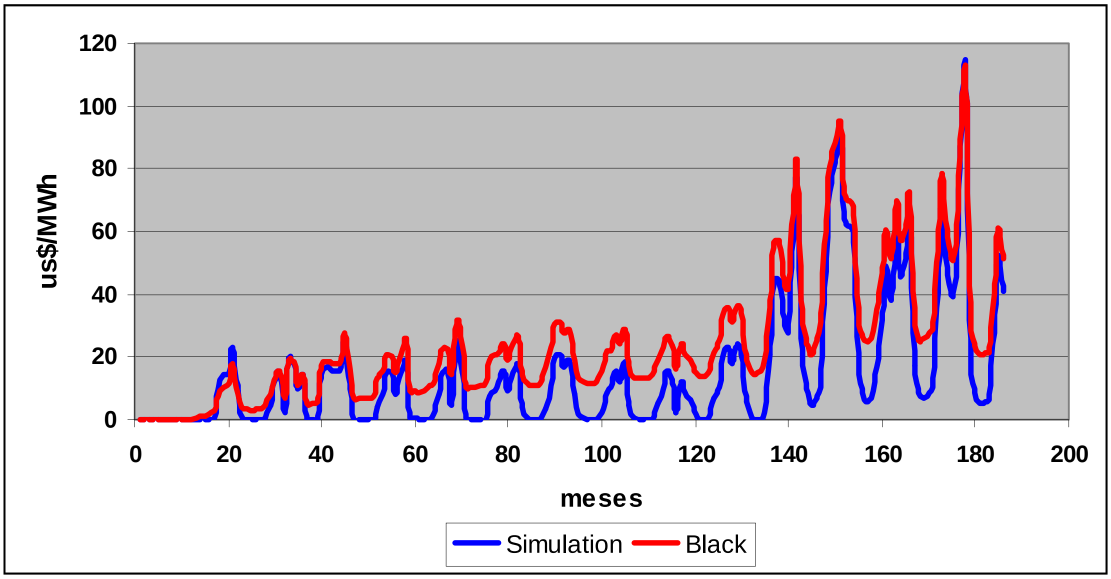
In the next figures we compare the estimation of the value of options using the result of market simulations and Black's formulae, for different exercise prices.

The (σ) volatility was estimated as 50%.

Comparison of Black's formulae with results of market simulation. Exercise price 30 us$/MWh. Volatility 50%.

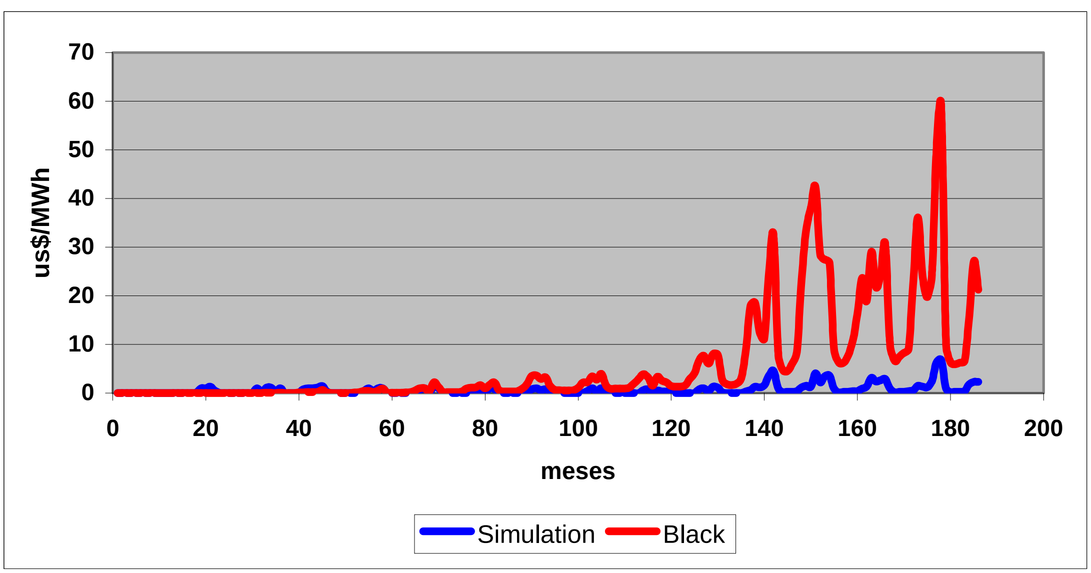


Comparison of Black's formulae with the results of market simulation. Exercise price 30 us$/MWh. Volatility 100%.



It is interesting to see that under higher volatility, the estimations of the value of options using Black's formulae are above the same estimations that result from the market simulations. Thus, Black's formulae overestimates the options prices when prices are low (high river discharges). This happens because Black's formulae assumes a constant volatility, while energy prices have lower dispersion in periods of high river discharges.

Comparison of Black's formulae with results of the market simulation. Exercise price 300 us$/MWh. Volatility 50%.



In this case, Black's formulae overestimates the options prices. But it is because energy prices have a cap established by the cost of unserved energy. In the market simulations we used a cost of unserved energy of 310 us$/MWh. Black's formulae cannot consider a price cap. Thus the important differences between the two methodologies can be justified.